6

Electromagnetic Induction

Multiple Choice Questions (MCQs)

Q. 1 A square of side L metres lies in the xy-plane in a region, where the magnetic field is given by $\mathbf{B} = B_0(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$ T, where B_0 is constant. The magnitude of flux passing through the square is

(a) $2B_0L^2$ Wb (b) $3B_0L^2$ Wb (c) $4B_0L^2$ Wb (d) $\sqrt{29}B_0L^2$ Wb

K **Thinking Process**

The magnetic flux linked with uniform surface of area A in uniform magnetic field is given by

$$\phi = \mathbf{B}.\mathbf{A}$$

Ans. *(c)* Here, $A = L^2\hat{\mathbf{k}}$ and $\mathbf{B} = B_0(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$ T

$$\phi = \mathbf{B}.\mathbf{A} = B_0(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}).L^2\hat{\mathbf{k}} = 4B_0L^2 \text{ Wb}$$

Q. 2 A loop, made of straight edges has six corners at A (0, 0, 0), B (L, 0, 0), C (L, L, 0), D (0, L, 0), E (0, L, L) and F (0, 0, L). A magnetic field $\mathbf{B} = B_0(\hat{\mathbf{i}} + \hat{\mathbf{k}})$ T is present in the region. The flux passing through the loop $ABCDEFA$ (in that order) is

(a) B_0L^2 Wb (b) $2B_0L^2$ Wb (c) $\sqrt{2}B_0L^2$ Wb (d) $4B_0L^2$ Wb

K **Thinking Process**

Here, loop ABCDA lies in x-y plane whose area vector $\mathbf{A}_1 = L^2\hat{\mathbf{k}}$ *whereas loop ADEFA lies in y-z plane whose area vector* $\mathbf{A}_2 = L^2\hat{\mathbf{i}}$.

Ans. *(b)* Also, the magnetic flux linked with uniform surface of area A in uniform magnetic field is given by

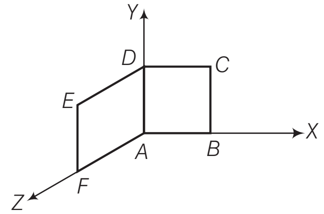
$$\phi = \mathbf{B}.\mathbf{A}$$

$$\mathbf{A} = \mathbf{A}_1 + \mathbf{A}_2 = (L^2\hat{\mathbf{k}} + L^2\hat{\mathbf{i}})$$

and $\mathbf{B} = B_0(\hat{\mathbf{i}} + \hat{\mathbf{k}})T$

Now, $\phi = \mathbf{B}.\mathbf{A} = B_0(\hat{\mathbf{i}} + \hat{\mathbf{k}}).(L^2\hat{\mathbf{k}} + L^2\hat{\mathbf{i}})$

$= 2B_0L^2$ Wb



Q. 3 A cylindrical bar magnet is rotated about its axis. A wire is connected from the axis and is made to touch the cylindrical surface through a contact. Then,

(a) a direct current flows in the ammeter *A*

(b) no current flows through the ammeter *A*

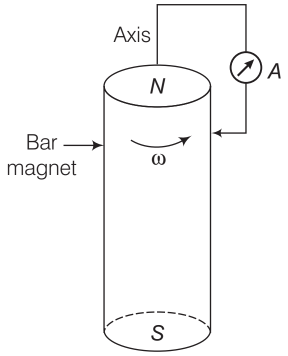
(c) an alternating sinusoidal current flows through the ammeter *A* with a time period $T = \frac{2\pi}{\omega}$

(d) a time varying non-sinusoidal current flows through the ammeter *A*

Thinking Process

The problem is associated with the phenomenon of electromagnetic induction.

Ans. *(b)* When cylindrical bar magnet is rotated about its axis, no change in flux linked with the circuit takes place, consequently no emf induces and hence, no current flows through the ammeter *A*.



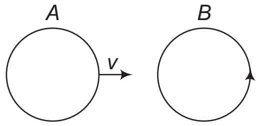
Q. 4 There are two coils *A* and *B* as shown in figure. *A* current starts flowing in *B* as shown, when *A* is moved towards *B* and stops when *A* stops moving. The current in *A* is counter clockwise. *B* is kept stationary when *A* moves. We can infer that

(a) there is a constant current in the clockwise direction in *A*

(b) there is a varying current in *A*

(c) there is no current in *A*

(d) there is a constant current in the counter clockwise direction in *A*



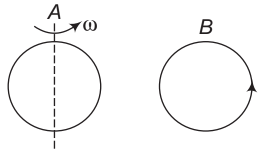
Thinking Process

The induced emf in B is due to the variation of magnetic flux in it.

Ans. *(d)* When the *A* stops moving the current in *B* become zero, it possible only if the current in *A* is constant. If the current in *A* would be variable, there must be an induced emf (current) in *B* even if the *A* stops moving.

Q. 5 Same as problem 4 except the coil A is made to rotate about a vertical axis (figure). No current flows in B if A is at rest. The current in coil A, when the current in B (at $t = 0$) is counter-clockwise and the coil A is as shown at this instant, $t = 0$, is

(a) constant current clockwise
(b) varying current clockwise
(c) varying current counter clockwise
(d) constant current counter clockwise



Thinking Process

Here, the application of Lenz's law is tested through this problem.

Ans. ***(a)*** When the current in B (at $t = 0$) is counter-clockwise and the coil A is considered above to it. The counterclockwise flow of the current in B is equivalent to north pole of magnet and magnetic field lines are emanating upward to coil A. When coil A start rotating at $t = 0$, the current in A is constant along clockwise direction by Lenz's rule.

Q. 6 The self inductance L of a solenoid of length l and area of cross-section A, with a fixed number of turns N increases as

(a) l and A increase
(b) l decreases and A increases
(c) l increases and A decreases
(d) both l and A decrease

Thinking Process

The self inductance L of a solenoid depends on its geometry (i.e., length, cross-sectional area, number of turns etc.) and on the permeability of the medium.

Ans. ***(b)*** The self-inductance of a long solenoid of cross-sectional area A and length l, having n turns per unit length, filled the inside of the solenoid with a material of relative permeability (*e.g.*, soft iron, which has a high value of relative permeability) is given by

$$L = \mu_r \mu_0 n^2 A l$$

where, $$n = N / l$$

Note *The capacitance, resistance, self and mutual inductance depends on the geometry of the devices as well as permittivity/permeability of the medium.*

Multiple Choice Questions (More Than One Options)

Q. 7 A metal plate is getting heated. It can be because

(a) a direct current is passing through the plate
(b) it is placed in a time varying magnetic field
(c) it is placed in a space varying magnetic field, but does not vary with time
(d) a current (either direct or alternating) is passing through the plate

K **Thinking Process**

This problem is associated with the heating effect of current as well as the phenomenon of electromagnetic induction and eddy currents.

Ans. *(a, b, d)*

A metal plate is getting heated when a DC or AC current is passed through the plate, known as heating effect of current. Also, when metal plate is subjected to time varying magnetic field, the magnetic flux linked with the plate changes and eddy currents comes into existence which make the plate hot.

Q. 8 An emf is produced in a coil, which is not connected to an external voltage source. This can be due to

(a) the coil being in a time varying magnetic field
(b) the coil moving in a time varying magnetic field
(c) the coil moving in a constant magnetic field
(d) the coil is stationary in external spatially varying magnetic field, which does not change with time

K **Thinking Process**

This problem is associated with the phenomenon of electromagnetic induction.

Ans. *(a, b, c)*

Here, magnetic flux linked with the isolated coil change when the coil being in a time varying magnetic field, the coil moving in a constant magnetic field or in time varying magnetic field.

Note *When magnetic flux linked with the coil change, an emf is used in the coil. This is known as electromagnetic induction.*

Q. 9 The mutual inductance M_{12} of coil 1 with respect to coil 2

(a) increases when they are brought nearer
(b) depends on the current passing through the coils
(c) increases when one of them is rotated about an axis
(d) is the same as M_{21} of coil 2 with respect to coil 1

K **Thinking Process**

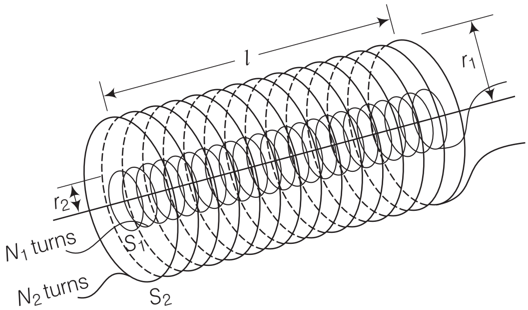
Here, it is important to know that the mutual inductance of a pair of coils, solenoids, etc., depends on their separation, their relative orientation as well as the geometry of pair of coils, solenoids, etc.

Ans. *(a, d)*

The mutual inductance M_{12} of coil increases when they are brought nearer and is the same as M_{21} of coil 2 with respect to coil 1.

M_{12} *i.e.,* mutual inductance of solenoid S_1 with respect to solenoid S_2 is given by

$$M_{21} = \mu_0 n_1 n_2 \pi r_1^2 l$$



where signs are as usual.

Also, M_{12} *i.e.*, mutual inductance of solenoid S_2 with respect to solenoid S_1 is given by

$$M_{21} = \mu_0 n_1 n_2 \pi r_1^2 l$$

So, we have $$M_{12} = M_{21} = M$$

Q. 10 A circular coil expands radially in a region of magnetic field and no electromotive force is produced in the coil. This can be because

(a) the magnetic field is constant

(b) the magnetic field is in the same plane as the circular coil and it may or may not vary

(c) the magnetic field has a perpendicular (to the plane of the coil) component whose magnitude is decreasing suitably

(d) there is a constant magnetic field in the perpendicular (to the plane of the coil) direction

Thinking Process

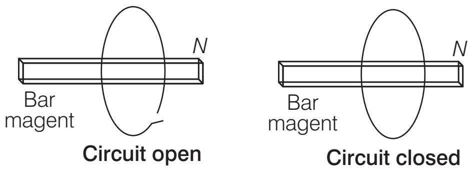
The various arrangement are to be thought of in such a way that the magnetic flux linked with the coil do not change even if coil is placed and expanding in magnetic field.

Ans. *(b, c)*

When circular coil expands radially in a region of magnetic field such that the magnetic field is in the same plane as the circular coil or the magnetic field has a perpendicular (to the plane of the coil) component whose magnitude is decreasing suitably in such a way that the cross product of magnetic field and surface area of plane of coil remain constant at every instant.

Very Short Answer Type Questions

Q. 11 Consider a magnet surrounded by a wire with an on/off switch S (figure). If the switch is thrown from the off position (open circuit) to the on position (closed circuit), will a current flow in the circuit? Explain.



K **Thinking Process**

The magnetic flux linked with uniform surface of area A in uniform magnetic field is given by

$$\phi = \mathbf{B}.\mathbf{A} = BA\cos\theta$$

So, flux linked will change only when either B, or A or the angle between B and A change.

Ans. When the switch is thrown from the off position (open circuit) to the on position (closed circuit), then neither B, nor A nor the angle between B and A change. Thus, no change in magnetic flux linked with coil occur, hence no electromotive force is produced and consequently no current will flow in the circuit.

Q. 12 A wire in the form of a tightly wound solenoid is connected to a DC source, and carries a current. If the coil is stretched so that there are gaps between successive elements of the spiral coil, will the current increase or decrease? Explain.

K **Thinking Process**

Here, the application of Lenz's law is tested through this problem.

Ans. When the coil is stretched so that there are gaps between successive elements of the spiral coil *i.e.,* the wires are pulled apart which lead to the flux leak through the gaps. According to Lenz's law, the emf produced must oppose this decrease, which can be done by an increase in current. So, the current will increase.

Q. 13 A solenoid is connected to a battery so that a steady current flows through it. If an iron core is inserted into the solenoid, will the current increase or decrease? Explain.

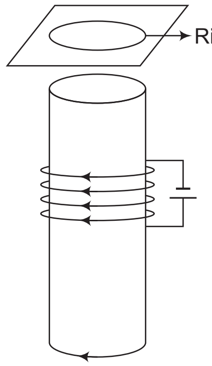
K **Thinking Process**

Here, the application of Lenz's law is tested through this problem.

Ans. When the iron core is inserted in the current carrying solenoid, the magnetic field increase due to the magnetisation of iron core and consequently the flux increases.

According to Lenz's law, the emf produced must oppose this increase in flux, which can be done by making decrease in current. So, the current will decrease.

Q. 14 Consider a metal ring kept on top of a fixed solenoid (say on a cardboard) (figure). The centre of the ring coincides with the axis of the solenoid. If the current is suddenly switched on, the metal ring jumps up. Explain



K **Thinking Process**

Here, the application of Lenz's law is tested through this problem.

Ans. When the current is switched on, magnetic flux is linked through the ring. Thus, increase in flux takes place. According to Lenz's law, this increase in flux will be opposed and it can happen if the ring moves away from the solenoid.

This happen because the flux increases will cause a counter clockwise current (as seen from the top in the ring in figure.) *i.e.,* opposite direction to that in the solenoid.

This makes the same sense of flow of current in the ring (when viewed from the bottom of the ring) and solenoid forming same magnetic pole infront of each other. Hence, they will repel each other and the ring will move upward.

Q. 15 Consider a metal ring kept (supported by a cardboard) on top of a fixed solenoid carrying a current I (see figure of Question 14). The centre of the ring coincides with the axis of the solenoid. If the current in the solenoid is switched off, what will happen to the ring?

K **Thinking Process**

This problem is based on the application of Lenz's law.

Ans. *(b)* When the current is switched off, magnetic flux linked through the ring decreases. According to Lenz's law, this decrease in flux will be opposed and the ring experience downward force towards the solenoid.

This happen because the flux *i* decrease will cause a clockwise current (as seen from the top in the ring in figure) *i.e.,* the same direction to that in the solenoid. This makes the opposite sense of flow of current in the ring (when viewed from the bottom of the ring) and solenoid forming opposite magnetic pole infront of each other.

Hence, they will attract each other but as ring is placed at the cardboard it could not be able to move downward.

Q. 16 Consider a metallic pipe with an inner radius of 1 cm. If a cylindrical bar magnet of radius 0.8 cm is dropped through the pipe, it takes more time to come down than it takes for a similar unmagnetised cylindrical iron bar dropped through the metallic pipe. Explain.

K **Thinking Process**

This problem is based on the concept of eddy current and application of Lenz's law.

Ans. When cylindrical bar magnet of radius 0.8 cm is dropped through the metallic pipe with an inner radius of 1 cm, flux linked with the cylinder changes and consequently eddy currents are produced in the metallic pipe. According to Lenz's law, these currents will oppose the (cause) motion of the magnet.

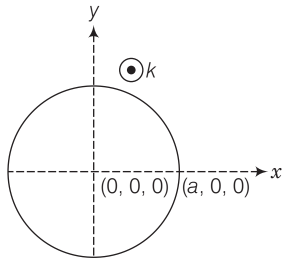
Therefore, magnet's downward acceleration will be less than the acceleration due to gravity g. On the other hand, an unmagnetised iron bar will not produce eddy currents and will fall with an acceleration due to gravity g.

Thus, the magnet will take more time to come down than it takes for a similar unmagnetised cylindrical iron bar dropped through the metallic pipe.

Short Answer Type Questions

Q. 17 A magnetic field in a certain region is given by $\mathbf{B} = B_0 \cos(\omega t)\,\hat{\mathbf{k}}$ and a coil of radius a with resistance R is placed in the x–y plane with its centre at the origin in the magnetic field (figure). Find the magnitude and the direction of the current at $(a, 0, 0)$ at

$$t = \frac{\pi}{2\omega},\ t = \frac{\pi}{\omega} \text{ and } t = \frac{3\pi}{2\omega}$$



K **Thinking Process**

This problem requires application of Faraday's law of EMI and finding mathematical values of emf at different instants.

Ans. At any instant, flux passes through the ring is given by

$$\phi = \mathbf{B}.\mathbf{A} = BA\cos\theta = BA \qquad (\because \theta = 0)$$

or
$$\phi = B_0(\pi a^2)\cos\omega t$$

By Faraday's law of electromagnetic induction.,

Magnitude of induced emf is given by

$$\varepsilon = B_0(\pi a^2)\,\omega\sin\omega t$$

This causes flow of induced current, which is given by

$$I = B_0(\pi a^2)\,\omega\sin\omega t\,/R$$

Now, finding the value of current at different instants, so we have current at

$$t = \frac{\pi}{2\omega}$$

$$I = \frac{B_0(\pi a^2)\omega}{R} \text{ along } \hat{\mathbf{j}}$$

Because $$\sin\omega t = \sin\left(\omega \frac{\pi}{2\omega}\right) = \sin\frac{\pi}{2} = 1$$

$$t = \frac{\pi}{\omega}, \; I = \frac{B(\pi a^2)\omega}{R}$$

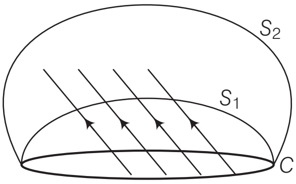
Here, $$\sin\omega t = \sin\left(\omega \frac{\pi}{\omega}\right) = \sin\pi = 0$$

$$t = \frac{3}{2}\frac{\pi}{\omega}$$

$$I = \frac{B(\pi a^2)\omega}{R} \text{ along } -\hat{\mathbf{j}}$$

$$\sin\omega t = \sin\left(\omega \frac{3\pi}{2\omega}\right) = \sin\frac{3\pi}{2} = -1$$

Q. 18 Consider a closed loop C in a magnetic field (figure). The flux passing through the loop is defined by choosing a surface whose edge coincides with the loop and using the formula $\phi = \mathbf{B}_1 d\mathbf{A}_1, \mathbf{B}_2 d\mathbf{A}_2$ ..... . Now, if we choose two different surfaces S_1 and S_2 having C as their edge, would we get the same answer for flux. Justify your answer.

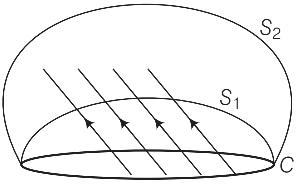


K **Thinking Process**

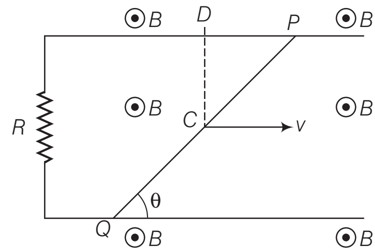
This problem underline the concept of continuity of magnetic field lines. They can neither be originated nor be destroyed in space.

Ans. The magnetic flux linked with the surface can considered as the number of magnetic field lines passing through the surface. So, let $d\phi = \mathbf{BA}$ represents magnetic lines in an area A to B.

By the concept of continuity of lines B cannot end or start in space, therefore the number of lines passing through surface S_1 must be the same as the number of lines passing through the surface S_2. Therefore, in both the cases we gets the same answer for flux.



Q. 19 Find the current in the wire for the configuration shown in figure. Wire PQ has negligible resistance. B, the magnetic field is coming out of the paper. θ is a fixed angle made by PQ travelling smoothly over two conducting parallel wires separated by a distance d.



K **Thinking Process**

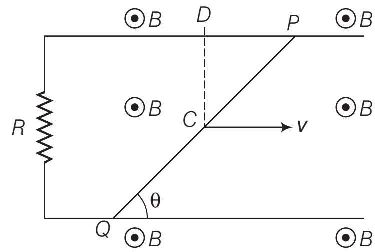
The emf induced across PQ due to its motion or change in magnetic flux linked with the loop change due to change of enclosed area.

Ans. The motional electric field E along the dotted line CD ($\perp$ to both **v** and **B** and along **V** × **B**) $= vB$

Therefore, the motional emf along PQ = (length PQ) × (field along PQ)

$$= (\text{length } PQ) \times (vB\sin\theta)$$

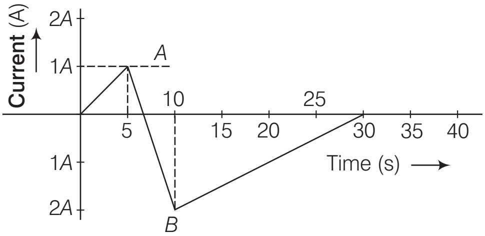
$$= \left(\frac{d}{\sin\theta}\right) \times (vB\sin\theta) = vBd$$



This induced emf make flow of current in closed circuit of resistance R.

$$I = \frac{dvB}{R} \text{ and is independent of } q.$$

Q. 20 A (current *versus* time) graph of the current passing through a solenoid is shown in figure. For which time is the back electromotive force (u) a maximum. If the back emf at $t = 3$ s is e, find the back emf at $t = 7$ s, 15s and 40 s. OA, AB and BC are straight line segments.



K **Thinking Process**

When the current is varied, the flux linked with the coil also changes and an emf is induced in the coil. The induced emf is given by

$$\varepsilon = -\frac{d(N\phi_B)}{dt}$$

$$\varepsilon = -L\frac{dI}{dt}$$

Thus, the self-induced emf always opposes any change (increase or decrease) of current in the coil.

Ans. The back electromotive force in solenoid is (u) a maximum when there is maximum rate of change of current. This occurs is in AB part of the graph. So maximum back emf will be obtained between $5\text{s} < t < 10\text{s}$.

Since, the back emf at $t = 3\text{s}$ is e,

Also,

the rate of change of current at $t = 3$, s = slope of OA from $t = 0\text{s}$ to $t = 5\text{s} = 1/5\text{ A/s}$.

So, we have

If $u = L\,1/5\left(\text{for } t = 3\text{s}, \frac{dI}{dt} = 1/5\right)$ (L is a constant). Applying $\varepsilon = -L\frac{dI}{dt}$

Similarly, we have for other values

For $5\text{s} < t < 10\text{s}$ $\quad u_1 = -L\frac{3}{5} = -\frac{3}{5}L = -3e$

Thus, $\quad$ at $t = 7\text{ s}$, $u_1 = -3e$

For $10\text{s} < t < 30\text{ s}$

$$u_2 = L\frac{2}{20} = \frac{L}{10} = \frac{1}{2}e$$

For $t > 30\text{ s}$, $u_2 = 0$

Thus, the back emf at $t = 7$ s, 15 s and 40 s are $-3e$, $e/2$ and 0 respectively.

Q. 21 **There are two coils A and B separated by some distance. If a current of 2A flows through A, a magnetic flux of 10^{-2} Wb passes through B (no current through B). If no current passes through A and a current of 1 A passes through B, what is the flux through A?**

K **Thinking Process**

A current I_1 is passed through the coil A and the flux linkage with coil B is,

$$N_2\phi_2 = M_{21}I_1$$

where, M_{21} is called the mutual inductance of coil A with respect to coil B and $M_{21} = M_{12}$

And M_{12} is called the mutual inductance of coil B with respect to coil A.

Ans. Applying the mutual inductance of coil A with respect to coil B

$$M_{21} = \frac{N_2\phi_2}{I_1}$$

Therefore, we have

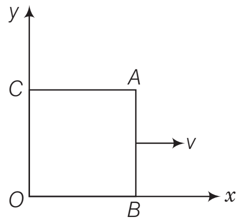
$$\text{Mutual inductance} = \frac{10^{-2}}{2} = 5\text{ mH}$$

Again applying this formula for other case

$$N_1\phi_1 = M_{12}I_2 = 5\text{ mH} \times 1\text{ A} = 5\text{ mWb}.$$

Long Answer Type Questions

Q. 22 A magnetic field $\mathbf{B} = B_0 \sin(\omega t)\hat{\mathbf{k}}$ covers a large region where a wire AB slides smoothly over two parallel conductors separated by a distance d (figure). The wires are in the x-y plane. The wire AB (of length d) has resistance R and the parallel wires have negligible resistance. If AB is moving with velocity v, what is the current in the circuit. What is the force needed to keep the wire moving at constant velocity?



K **Thinking Process**

The emf induced across AB due to its motion and change in magnetic flux linked with the loop change due to change of magnetic field.

Ans. Let us assume that the parallel wires at are $y = 0$ *i.e.,* along x-axis and $y = d$. At $t = 0$, AB has $x = 0$, *i.e.,* along y-axis and moves with a velocity v. Let at time t, wire is at $x(t) = vt$.

Now, the motional emf across AB is

$$= (B_0 \sin\omega t) vd(-\hat{\mathbf{j}})$$

emf due to change in field (along $OBAC$)

$$= -B_0 \omega \cos\omega t \, x(t) \, d$$

Total emf in the circuit = emf due to change in field (along $OBAC$) + the motional emf across AB

$$= -B_0 d[\omega x \cos(\omega t) + v \sin(\omega t)]$$

Electric current in clockwise direction is given by

$$= \frac{B_0 d}{R}(\omega x \cos\omega t + v \sin\omega t)$$

The force acting on the conductor is given by $F = ilB \sin 90° = ilB$

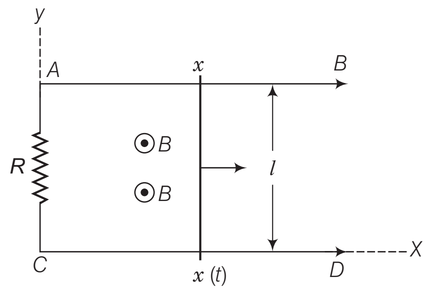
Substituting the values, we have

$$\text{Force needed along } \mathbf{i} = \frac{B_0 d}{R}(\omega x \cos\omega t + v \sin\omega t) \times d \times B_0 \sin\omega t$$

$$= \frac{B_0^2 d^2}{R}(\omega x \cos\omega t + v \sin\omega t) \sin\omega t$$

This is the required expression for force.

Q. 23 A conducting wire XY of mass m and negligible resistance slides smoothly on two parallel conducting wires as shown in figure. The closed circuit has a resistance R due to AC. AB and CD are perfect conductors. There is a magnetic field $\mathbf{B} = B(t)\hat{\mathbf{k}}$



(i) Write down equation for the acceleration of the wire XY.

(ii) If **B** is independent of time, obtain $v(t)$, assuming $v(0) = u_0$

(iii) For (ii), show that the decrease in kinetic energy of XY equals the heat lost in .

K **Thinking Process**

This problem relates EMI, magnetic force, power consumption and mechanics.

Ans. Let us assume that the parallel wires at are $y = 0$, *i.e.*, along x-axis and $y = l$. At $t = 0$, XY has $x = 0$ *i.e.*, along y-axis.

(i) Let the wire be at $x = x(t)$ at time t.

The magnetic flux linked with the loop is given by

$$\phi = \mathbf{B.A} = BA\cos 0 = BA$$

at any instant t Magnetic flux $= B(t)(l \times x(t))$

Total emf in the circuit = emf due to change in field (along $XYAC$) + the motional emf across XY

$$E = -\frac{d\phi}{dt} = -\frac{dB(t)}{dt} lx(t) - B(t)lv(t) \qquad \text{[second term due to motional emf]}$$

Electric current in clockwise direction is given by

$$I = \frac{1}{R}E$$

The force acting on the conductor is given by $F = ilB\sin 90° = ilB$

Substituting the values, we have

$$\text{Force} = \frac{IB(t)}{R}\left[-\frac{dB(t)}{dt} I\, x(t) - B(t)Iv(t)\right]\hat{\mathbf{i}}$$

Applying Newton's second law of motion,

$$m\frac{d^2x}{dt^2} = -\frac{I^2B(t)}{R}\frac{dB}{dt}x(t) - \frac{I^2B^2(t)}{R}\frac{dx}{dt} \qquad \text{...(i)}$$

which is the required equation.

(ii) If **B** is independent of time *i.e.*, $B = \text{Constant}$

Or $$\frac{dB}{dt} = 0$$

Substituting the above value in Eq (i), we have

$$\frac{d^2x}{dt^2} + \frac{I^2B^2}{mR}\frac{dx}{dt} = 0$$

or

$$\frac{dv}{dt} + \frac{I^2B^2}{mR}v = 0$$

Integrating using variable separable form of differential equation, we have

$$v = A\exp\left(\frac{-I^2B^2t}{mR}\right)$$

Applying given conditions, at $t = 0$, $v = u_0$

$$v(t) = u_0 \exp(-I^2B^2t / mR)$$

This is the required equation.

(iii) Since the power consumption is given by $P = I^2R$

Here,

$$I^2R = \frac{B^2I^2v^2(t)}{R^2} \times R$$

$$= \frac{B^2I^2}{R}u_0^2 \exp(-2I^2B^2t\ I\ mR)$$

Now, energy consumed in time interval dt is given by energy consumed $= Pdt = I^2Rdt$

Therefore, total energy consumed in time t

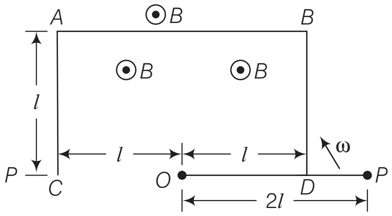
$$= \int_0^t I^2Rdt = \frac{B^2I^2}{R}u_0^2 \frac{mR}{2I^2B^2}\left[1 - e^{-(l^2B^2t/mr)}\right]$$

$$= \frac{m}{2}u_0^2 - \frac{m}{2}v^2(t)$$

$=$ decrease in kinetic energy.

This proves that the decrease in kinetic energy of XY equals the heat lost in R.

Q. 24 *ODBAC* is a fixed rectangular conductor of negligible resistance (*CO* is not connected) and *OP* is a conductor which rotates clockwise with an angular velocity ω (figure). The entire system is in a uniform magnetic field **B** whose direction is along the normal to the surface of the rectangular conductor *ABDC*. The conductor *OP* is in electric contact with *ABDC*. The rotating conductor has a resistance of λ per unit length. Find the current in the rotating conductor, as it rotates by 180°.

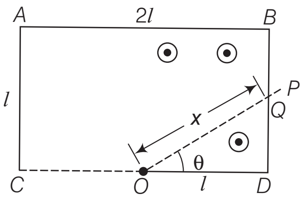


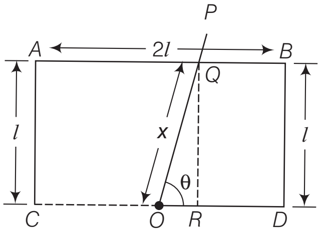
K **Thinking Process**

The pattern of rate of change of area (hence flux) can be considered uniform from $0 < \theta < \frac{\pi}{4}$; $\frac{\pi}{4} < \theta < \frac{3\pi}{4}$ *and* $\frac{3\pi}{4} < \theta < \frac{\pi}{2}$. *Hence, forth finding emf and current.*

Ans. Let us consider the position of rotating conductor at time interval

$$t = 0 \text{ to } t = \frac{\pi}{4\omega} \text{ (or } T/8)$$





the rod OP will make contact with the side BD. Let the length OQ of the contact at sometime t such that $0 < t < \frac{\pi}{4\omega}$ or $0 < t < \frac{T}{8}$ be x. The flux through the area ODQ is

$$\phi = B\frac{1}{2}QD \times OD = B\frac{1}{2}l\tan\theta \times l$$

$$= \frac{1}{2}Bi^2\tan\theta, \text{ where } \theta = \omega t$$

Applying Faraday's law of EMI,

Thus, the magnitude of the emf generated is $\varepsilon = \frac{d\phi}{dt} = \frac{1}{2}Bl^2\omega\sec^2\omega t$

The current is $I = \frac{\varepsilon}{R}$ where R is the resistance of the rod in contact.

where, $R \propto \lambda$

$$R = \lambda x = \frac{\lambda l}{\cos\omega t}$$

$\therefore$
$$I = \frac{1}{2}\frac{Bl^2\omega}{\lambda l}\sec^2\omega t\cos\omega t = \frac{Bl\omega}{2\lambda\cos\omega t}$$

Let the length OQ of the contact at some time t such that $\frac{\pi}{4\omega} < t < \frac{3\pi}{4\omega}$ or $\frac{T}{8} < t < \frac{3T}{8}$ be x. The rod is in contact with the side AB. The flux through the area $OQBD$ is

$$\phi = \left(l^2 + \frac{1}{2}\frac{l^2}{\tan\theta}\right)B$$

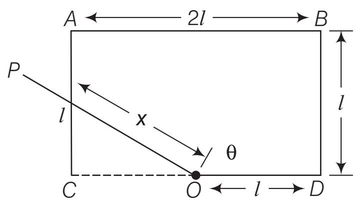
Where, $\theta = \omega t$

Thus, the magnitude of emf generated in the loop is

$$\varepsilon = \frac{d\phi}{dt} = \frac{1}{2}Bl^2\omega\frac{\sec^2\omega t}{\tan^2\omega t}$$

The current is $I = \frac{\varepsilon}{R} = \frac{\varepsilon}{\lambda x} = \frac{\varepsilon\sin\omega t}{\lambda l} = \frac{1}{2}\frac{Bl\omega}{\lambda\sin\omega t}$

Similarly for $\frac{3\pi}{4\omega} < t < \frac{\pi}{\omega}$ or $\frac{3T}{8} < t < \frac{T}{2}$, the rod will be in touch with AC.



The flux through *OQABD* is given by

$$\phi = \left(2l^2 - \frac{l^2}{2\tan\omega t}\right)B$$

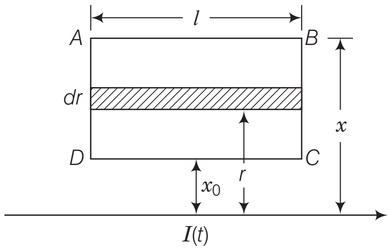
And the magnitude of emf generated in loop is given by

$$\varepsilon = \frac{d\phi}{dt} = \frac{B\omega l^2 \sec^2 \omega t}{2\tan^2 \omega t}$$

$$I = \frac{\varepsilon}{R} = \frac{\varepsilon}{\lambda x} = \frac{1}{2}\frac{Bl\omega}{\lambda \sin\omega t}$$

These are the required expressions.

Q. 25 Consider an infinitely long wire carrying a current $I(t)$, with $\frac{dI}{dt} = \lambda$ = constant. Find the current produced in the rectangular loop of wire *ABCD* if its resistance is *R* (figure).



K **Thinking Process**

This question need the use of integration in order to find the total magnetic flux linked with the loop.

Ans. Let us consider a strip of length l and width dr at a distance r from infinite long current carrying wire. The magnetic field at strip due to current carrying wire is given by

$$\text{Field } B(r) = \frac{\mu_0 I}{2\pi r} \text{ (out of paper)}$$

Total flux through the loop is

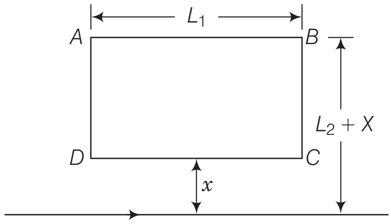
$$\text{Flux} = \frac{\mu_0 I}{2\pi} l \int_{x_0}^{x} \frac{dr}{r} = \frac{\mu_0 I}{2\pi} \ln\frac{x}{x_0} \quad \text{...(i)}$$

The emf induced can be obtained by differentiating the eq. (i) wrt t and then applying Ohm's law

$$\frac{\varepsilon}{R} = I$$

We have, $\text{induced current} = \frac{1}{R}\frac{d\phi}{dt} = \frac{\varepsilon}{R} = \frac{\mu_0 I}{2\pi}\frac{\lambda}{R}\ln\frac{x}{x_0}$ $\quad \left(\because \frac{dI}{dt} = \lambda\right)$

Q. 26 A rectangular loop of wire $ABCD$ is kept close to an infinitely long wire carrying a current $I(t) = I_0(1 - t/T)$ for $0 \le t \le T$ and $I(0) = 0$ for $t > T$ (figure.). Find the total charge passing through a given point in the loop, in time T. The resistance of the loop is R.



Thinking Process

The charge passes through the circuit can be obtained by finding the relation between instantaneous current and instantaneous magnetic flux linked with it.

Ans. The emf induced can be obtained by differentiating the expression of magnetic flux linked wrt t and then applying Ohm's law

$$I = \frac{E}{R} = \frac{1}{R}\frac{d\phi}{dt}$$

We know that electric current

$$I(t) = \frac{dQ}{dt} \quad \text{or} \quad \frac{dQ}{dt} = \frac{1}{R}\frac{d\phi}{dt}$$

Integrating the variable separable form of differential equation for finding the charge Q that passed in time t, we have

$$Q(t_1) - Q(t_2) = \frac{1}{R}[\phi(t_1) - \phi(t_2)]$$

$$\phi(t_1) = L_1 \frac{\mu_0}{2\pi}\int_x^{L_2 + x}\frac{dx'}{x'} I(t_1) \qquad \text{[Refer to the Eq. (i) of answer no.25]}$$

$$= \frac{\mu_0 L_1}{2\pi} I(t_1) \ln\frac{L_2 + x}{x}$$

The magnitude of charge is given by,

$$= \frac{\mu_0 L_1}{2\pi}\ln\frac{L_2 + x}{x}[I_0 + 0]$$

$$= \frac{\mu_0 L_1}{2\pi} I_1 \ln\left(\frac{L_2 + x}{x}\right)$$

This is the required expression.

Q. 27 A magnetic field $\mathbf{B}$ is confined to a region $r \le a$ and points out of the paper (the z-axis), $r = 0$ being the centre of the circular region. A charged ring (charge $= Q$) of radius b, $b > a$ and mass m lies in the x-y plane with its centre at the origin. The ring is free to rotate and is at rest. The magnetic field is brought to zero in time Δt. Find the angular velocity ω of the ring after the field vanishes.

Thinking Process

The decrease in magnetic field causes induced emf and hence, electric field around ring. The torque experienced by the ring produces change in angular momentum.

Ans. Since, the magnetic field is brought to zero in time Δt, the magnetic flux linked with the ring also reduces from maximum to zero. This, in turn, induces an emf in ring by the phenomenon of EMI. The induces emf causes the electric field E generation around the ring.

The induced emf = electric field $E \times (2\pi b)$ (Because $V = E \times d$) ...(i)
By Faraday's law of EMI
The induced emf = rate of change of magnetic flux

= rate of change of magnetic field $\times$ area

$$= \frac{B\pi a^2}{\Delta t} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we have

$$2\pi bE = emf = \frac{B\pi a^2}{\Delta t}$$

Since, the charged ring experienced a electric force = QE
This force try to rotate the coil, and the torque is given by

$$\text{Torque} = b \times \text{Force}$$

$$= QEb = Q\left[\frac{B\pi a^2}{2\pi b\Delta t}\right]b$$

$$= Q\frac{Ba^2}{2\Delta t}$$

If ΔL is the change in angular momentum

$$\Delta L = \text{Torque} \times \Delta t = Q\frac{Ba^2}{2}$$

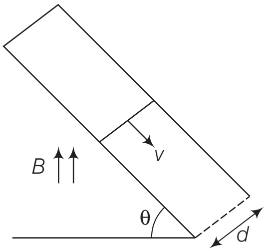
Since, initial angular momentum = 0
Now, since Torque $\times \Delta t$ = Change in angular momentum

Final angular momentum $= mb^2\omega = \dfrac{QBa^2}{2}$

$$\omega = \frac{QBa^2}{2mb^2}$$

On rearranging the terms, we have the required expression of angular speed.

Q. 28 A rod of mass m and resistance R slides smoothly over two parallel perfectly conducting wires kept sloping at an angle θ with respect to the horizontal (figure). The circuit is closed through a perfect conductor at the top. There is a constant magnetic field **B** along the vertical direction. If the rod is initially at rest, find the velocity of the rod as a function of time.



K **Thinking Process**

This problem combines the mechanics, EMI, magnetic force and linear differential equation.

Ans. Here, the component of magnetic field perpendicular the plane = $B\cos\theta$
Now, the conductor moves with speed v perpendicular to $B\cos\theta$ component of magnetic field. This causes motional emf across two ends of rod, which is given by = $v(B\cos\theta)d$

This makes flow of induced current $i = \frac{v(B\cos\theta)d}{R}$ where, R is the resistance of rod. Now, current carrying rod experience force which is given by $F = iBd$ (horizontally in backward direction). Now, the component of magnetic force parallel to incline plane along upward direction $= F\cos\theta = iBd\cos\theta = \left(\frac{v(B\cos\theta)d}{R}\right)Bd\cos\theta$ where, $v = \frac{dx}{dt}$

Also, the component of weight (mg) parallel to incline plane along downward direction $= mg\sin\theta$.

Now, by Newton's second law of motion

$$m\frac{d^2x}{dt^2} = mg\sin\theta - \frac{B\cos\theta d}{R}\left(\frac{dx}{dt}\right) \times (Bd)\cos\theta$$

$$\frac{dv}{dt} = g\sin\theta - \frac{B^2d^2}{mR}(\cos\theta)^2 v$$

$$\frac{dv}{dt} + \frac{B^2d^2}{mR}(\cos\theta)^2 v = g\sin\theta$$

But, this is the linear differential equation.

On solving, we get

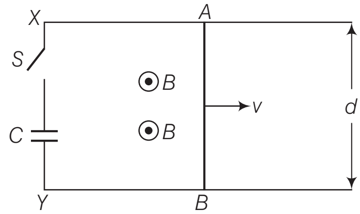
$$v = \frac{g\sin\theta}{\frac{B^2d^2\cos^2\theta}{mR}} + A\exp\left(-\frac{B^2d^2}{mR}(\cos^2\theta)t\right)$$

A is a constant to be determine by initial conditions.

The required expression of velocity as a function of time is given by

$$= \frac{mgR\sin\theta}{B^2d^2\cos^2\theta}\left(1 - \exp\left(-\frac{B^2d^2}{mR}(\cos^2\theta)t\right)\right)$$

Q. 29 Find the current in the sliding rod AB (resistance $= R$) for the arrangement shown in figure. B is constant and is out of the paper. Parallel wires have no resistance, v is constant. Switch S is closed at time $t = 0$.



K **Thinking Process**

This problem combines the concept of EMI, charging of capacitor and linear differential equation.

Ans. The conductor of length d moves with speed v, perpendicular to magnetic field B as shown in figure. This produces motional emf across two ends of rod, which is given by $= vBd$. Since, switch S is closed at time $t = 0$. capacitor is charged by this potential difference. Let $Q(t)$ is charge on the capacitor and current flows from A to B.

Now, the induced current

$$I = \frac{vBd}{R} - \frac{Q}{RC}$$

On rearranging the terms, we have

$$\frac{Q}{RC} + \frac{dQ}{dt} = \frac{vBd}{R}$$

This is the linear differential equation. On solving, we get

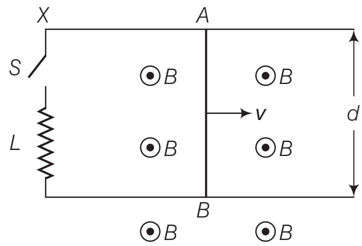
$$Q = vBdC + Ae^{-t/RC}$$

$$\Rightarrow \quad Q = vBdC\,[1 - e^{-t/RC}] \qquad (\text{At time } t = 0,\ Q = 0 = A = -\,vBdc).$$

Differentiating, we get $I - \frac{vBd}{R} e^{-t/RC}$

This is the required expression of current.

Q. 30 Find the current in the sliding rod *AB* (resistance = *R*) for the arrangement shown in figure. **B** is constant and is out of the paper. Parallel wires have no resistance, **v** is constant. Switch *S* is closed at time $t = 0$.



K **Thinking Process**

This problem combines the concept of EMI, growth of current in inductor and linear differential equation.

Ans. The conductor of length *d* moves with speed *v*, perpendicular to magnetic field **B** as shown in figure. This produces motional emf across two ends of rod, which is given by $= vBd$.

Since, switch *S* is closed at time $t = 0$. current start growing in inductor by the potential difference due to motional emf.

Applying Kirchhoff's voltage rule, we have

$$-L\frac{dI}{dt} + vBd = IR \quad \text{or} \quad L\frac{dI}{dt} + IR = vBd$$

This is the linear differential equation. On solving, we get

$$I = \frac{vBd}{R} + Ae^{-Rt/2}$$

$$\text{At } t = 0 \quad I = 0$$

$$\Rightarrow \quad A = -\frac{vBd}{R} \Rightarrow I = \frac{vBd}{R}(1 - e^{-Rt/L}).$$

This is the required expression of current.

Q. 31 A metallic ring of mass *m* and radius *l* (ring being horizontal) is falling under gravity in a region having a magnetic field. If *z* is the vertical direction, the *z*-component of magnetic field is $B_z = B_0\,(1 + \lambda z)$. If *R* is the resistance of the ring and if the ring falls with a velocity *v*, find the energy lost in the resistance. If the ring has reached a constant velocity, use the conservation of energy to determine *v* in terms of *m*, *B*, λ and acceleration due to gravity *g*.

K **Thinking Process**

This problem establishes a relationship between induced current, power lost and velocity acquired by freely falling ring.

Ans. The magnetic flux linked with the metallic ring of mass m and radius l falling under gravity in a region having a magnetic field whose z-component of magnetic field is $B_z = B_0 (1 + \lambda z)$ is

$$\phi = B_z (\pi l^2) = B_0 (1 + \lambda z)(\pi l^2)$$

Applying Faraday's law of EMI, we have emf induced given by $\frac{d\phi}{dt}$ = rate of change of flux

Also, by Ohm's law

$$B_0 (\pi l^2)\lambda \frac{dz}{dt} = IR$$

On rearranging the terms, we have $$I = \frac{\pi l^2 B_0 \lambda}{R} v$$

$$\text{Energy lost/second} = I^2 R = \frac{(\pi l^2 \lambda)^2 B_0^2 v^2}{R}$$

This must come from rate of change in PE $= mg \frac{dz}{dt} = mgv$

[as kinetic energy is constant for $v =$ constant]

Thus, $$mgv = \frac{(\pi l^2 \lambda B_0)^2 v^2}{R} \quad or \quad v = \frac{mgR}{(\pi l^2 \lambda B_0)^2}$$

This is the required expression of velocity.

Q. 32 A long solenoid S has n turns per meter, with diameter a. At the centre of this coil, we place a smaller coil of N turns and diameter b (where $b < a$). If the current in the solenoid increases linearly, with time, what is the induced emf appearing in the smaller coil. Plot graph showing nature of variation in emf, if current varies as a function of $mt^2 + C$.

K **Thinking Process**

This problem require an insight to magnetic field due to current carrying solenoid having varying current which induces emf in coil of radius B.

Ans. Magnetic field due to a solenoid S, $B = \mu_0 n l$ where signs are as usual.

Magnetic flux in smaller coil $\phi = NBA$, where

$$A = \pi b^2$$

Applying Faraday's law of EMI, we have

So, $$e = \frac{-d\phi}{dt} = \frac{-d}{dt}(NBA)$$

$$= -N\pi b^2 \frac{d(B)}{dt}$$

where, $$B = \mu_0 Ni$$

$$= -N\pi b^2 \mu_0 n \frac{dI}{dt}$$

$$= -Nn\pi\mu_0 b^2 \frac{d}{dt}(mt^2 + C) = -\mu_0 Nn\pi b^2 2mt$$

Since, current varies as a function of $mt^2 + C$.

$$e = -\mu_0 Nn\pi b^2 2mt$$